On local spectra preserver problems

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- Let $\mathcal{A}$ and $\mathcal{B}$ be semisimple Banach algebras and $\phi : \mathcal{A} \to \mathcal{B}$ linear, unital and surjective such that

$$\sigma(\phi(x)) \subseteq \sigma(x) \quad (\forall x \in \mathcal{A}). \tag{1}$$

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The local spectrum at some vector

- For $T \in \mathcal{L}(X)$, its local resolvent set at $x \in X$ is the union of all open subsets $U \subseteq \mathbf{C}$ for which there exists an analytic function $f : U \to X$ such that $(T - \lambda I) f(\lambda) = x$ for all $\lambda \in U$. The local spectrum of T at x, denoted by $\sigma_T(x)$, is defined as the complement in $\mathbf{C}$ of the local resolvent set of T at x.

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- **Theorem (A. Bourhim and T. Ransford, Int. Eq. Oper. Th., 2005)** Let $\varphi : \mathcal{L}(X) \to \mathcal{L}(X)$ be an additive map such that

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Then $\varphi(T) = T$ for all $T \in \mathcal{L}(X)$.

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  ii) If
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Preservers of local spectrum/spectral radius

- Ideea: the set of all operators $T \in \mathcal{L}(X)$ such that the surjectivity spectrum of T coincides with $\sigma_T(x_0)$ is dense in $\mathcal{L}(X)$. As a corollary, we obtain that the set of all $T \in \mathcal{L}(X)$ such that $r_T(x_0) = \rho(T)$ is also dense. Then using the continuity hypothesis we have $\rho(\varphi(T)) = \rho(T)$ for all $T \in \mathcal{L}(X)$. The surjective spectral isometries of $\mathcal{L}(X)$ are of a standard form!

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- For a closed subset $F \subseteq \mathbf{C}$ and an operator $T \in \mathcal{L}(X)$, we defined the glocal spectral subspace of T as

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Automatic continuity

- **Theorem (–, 2017).** Fix a nonzero vector $x_0 \in X$ and let $\varphi : \mathcal{L}(X) \to \mathcal{L}(X)$ be a linear surjective map such that

$$\Gamma_{\varphi(T)}(x_0) \leq \rho(T) \qquad (T \in \mathcal{L}(X)).$$

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- **Corollary.** Let $\varphi : \mathcal{L}(X) \to \mathcal{L}(X)$ be a linear surjective map such that

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Preservers of inner/outer local spectral radius

- **Theorem (–, 2018).** Let $\varphi : \mathcal{L}(X) \to \mathcal{L}(X)$ be a linear surjective map such that

$$\sigma_{\varphi(T)}(x) \cap \sigma_T(x) \neq \emptyset$$

for each $T \in \mathcal{L}(X)$ and $x \in X$ such that at least one of the above sets is nonempty. Then φ is the identity of $\mathcal{L}(X)$.

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- **Theorem (M. González and M. Mbekhta, Linear Algebra Appl., 2007).** Fix a nonzero vector $x_0 \in \mathbf{C}^n$ and let $\varphi : \mathcal{M}_n \to \mathcal{M}_n$ be a linear map such that

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There exists then an invertible matrix $A \in \mathcal{M}_n$ such that $Ax_0 = x_0$ and

$$\varphi(T) = ATA^{-1} \qquad (T \in \mathcal{M}_n).$$

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- **Theorem (A. Bourhim and –, 2018).** For a nonzero fixed vector $x_0 \in \mathbf{C}^2$, a linear map φ on $\mathcal{M}_2$ satisfies

$$r_T(x_0) = 0 \Longleftrightarrow r_{\varphi(T)}(x_0) = 0 \qquad (T \in \mathcal{M}_2)$$

if and only if there exists a nonzero scalar α, an invertible matrix $U \in \mathcal{M}_2$ for which $Ux_0 = x_0$ and a matrix $Q \in \mathcal{M}_2$ satisfying $Qx_0 = 0$ and $\mathrm{tr}(Q) \neq -1$ such that

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- **Theorem (A. Bourhim and –, 2018).** Let $n \geq 3$ be a natural number, and fix a nonzero vector $x_0 \in \mathbf{C}^n$. A linear map $\varphi : \mathcal{M}_n \to \mathcal{M}_n$ satisfies

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Maps preserving matrices of inner local spectral radius zero at some fixed vector

- **Theorem (–, 2018).** Let $n \geq 2$ be a natural number. Let $x_0 \in \mathbb{C}^n$ be a fixed nonzero vector and let $\varphi : \mathcal{M}_n \to \mathcal{M}_n$ be a surjective additive map. Then

$$i_T(x_0) = 0 \Longrightarrow i_{\varphi(T)}(x_0) = 0 \qquad (T \in \mathcal{M}_n) \qquad (5)$$

if and only if there exist a nonzero c, a field automorphism $\eta : \mathbf{C} \to \mathbf{C}$, an invertible matrix $A \in \mathcal{M}_n$ satisfying $A(x_0^\eta) = x_0$ and a vector $f \in \mathbf{C}^n$ satisfying $f^t x_0 \neq 1$ such that

$$\varphi(T) = cA(T - x_0 f^t T)^\eta A^{-1} \qquad (T \in \mathcal{M}_n). \qquad (6)$$

We arrive at the same conclusion by supposing

$$i_{\varphi(T)}(x_0) = 0 \Rightarrow i_T(x_0) = 0 \qquad (T \in \mathcal{M}_n) \qquad (7)$$

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Thank You!