Résumé:

For a locally compact group G we have pairs of algebras $L1(G)$, the group algebra, and $A(G)$, the Fourier algebra; and $M(G)$, the measure algebra, and $B(G)$, the Fourier-Stieltjes algebra. These pairs are each "dual pairs" in a sense which generalises Pontryagin duality from abelian groups. The realisation that $A(G)$ and $B(G)$ admit certain natural operator space structures augments our understanding of the duality. B. Johnson's now classic theorem is that $L1(G)$ is an amenable Banach algebra, if and only if G is an amenable group. The dual version is Z.-J. Ruan's theorem that $A(G)$ is an operator amenable completely contractive Banach algebra, if and only if G is an amenable. Also, Johnson proved that $L1(G)$ is always weakly amenable; while I, and independently E. Samei, proved that $A(G)$ is always operator weakly amenable.

It was proved recently by Dales, Ghahramani and Helemskii that $M(G)$ is weakly amenable if and only if G is discrete; and further that $M(G)$ is amenable if and only if G is discrete and amenable. The natural conjecture is that the dual result must hold: $B(G)$ *is operator (weakly) amenable if and only if* G *is compact.* V. Runde and I had produced evidence suggesting that this conjecture is true. However, we now have a counterexample to show it is false in a more profound way than we expected.