

Arithmetic complexity of Erdős matrices

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Abstract

Erdős matrices are the doubly stochastic matrices for which equality holds in the Marcus–Ree inequality

$$\max \text{trace}(M) \geq \|M\|_{\mathbb{F}}^2.$$

Every such matrix has rational entries, but little is known about the growth of the denominators that occur. In this talk, we will discuss the study of this arithmetic complexity. Using a block-diagonal construction involving the matrices T_m (zero diagonal, constant off-diagonal) for prime sizes, we will present a proof that the largest denominator D_n of an $n \times n$ Erdős matrix satisfies

$$D_n \geq \exp(c\sqrt{n \log n})$$

for some constant $c > 0$ and all $n \geq 1$. On the other hand, by relating an Erdős matrix to the Laplacian of its bipartite support graph and applying the Matrix-Tree Theorem, we will explain how to establish the upper bound

$$D_n \leq n^{2n} = \exp(2n \log n).$$

We will also mention that the Gram matrix associated with a Birkhoff decomposition admits a natural representation-theoretic decomposition into trivial and standard components.