Cantor sets and cyclicity in weighted Dirichlet spaces

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Abstract

We treat the problem of characterizing the cyclic vectors in the weighted Dirichlet spaces, extending some of our earlier results in the classical Dirichlet space. The absence of a Carleson-type formula for weighted Dirichlet integrals necessitates the introduction of new techniques.

Keywords: Dirichlet space, weight, cyclic vector, α-capacity, Cantor set
2000 MSC: 30H05, 46E20, 47A15

1. Introduction

In this paper we study the weighted Dirichlet spaces $\mathcal{D}_\alpha$ $(0 \le \alpha \le 1)$, defined by

$$\mathcal{D}_\alpha := \Big\{ f \in \mathrm{hol}(\mathbb{D}) : \mathcal{D}_\alpha(f) := \frac{1}{\pi} \int_{\mathbb{D}} |f'(z)|^2 (1-|z|^2)^\alpha \, dA(z) < \infty \Big\}.$$

Here $\mathbb{D}$ denotes the open unit disk, and dA is area measure on $\mathbb{D}$. Clearly $\mathcal{D}_\alpha$ is a Hilbert space with respect to the norm $\|\cdot\|_\alpha$ given by

$$\|f\|_\alpha^2 := |f(0)|^2 + \mathcal{D}_\alpha(f).$$

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