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ADMISSIBLE FUNCTIONS FOR THE DIRICHLET SPACE

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Abstract. Zero sets and uniqueness sets of the classical Dirichlet space $\mathcal{D}$ are not completely characterized yet. We define the concept of admissible functions for the Dirichlet space and then apply them to obtain a new class of zero sets for $\mathcal{D}$. Then we discuss the relation between the zero sets of $\mathcal{D}$ and those of $\mathcal{A}^\infty$.

1. Introduction

Let $f(z) = \sum_{n=0}^\infty a_n z^n$ be holomorphic on the open unit disk $\mathbb{D}$. Then, by direct verification, we obtain

$$\mathcal{D}(f) := \frac{1}{\pi}\int_{\mathbb{D}} |f'(z)|^2\, dA(z) = \sum_{n=1}^\infty n\, |a_n|^2,$$

where dA is the two-dimensional Lebesgue measure. The *Dirichlet space* is by definition

$$\mathcal{D} = \{\, f \in \mathrm{Hol}(\mathbb{D}) \,:\, \mathcal{D}(f) < \infty \,\}.$$

It is clear that the classical Hardy space $H^2(\mathbb{D})$ contains the Dirichlet space $\mathcal{D}$ as a proper subclass. Considering the norm

$$\|f\|_{\mathcal{D}}^2 = \mathcal{D}(f) + \|f\|_{H^2}^2,$$

the Dirichlet space becomes a Hilbert space of analytic functions on the open unit disc whose inner product is given by

$$\langle f, g\rangle = \sum_{n=0}^\infty (n+1)\, a_n\, \bar{b}_n,$$

where $f(z) = \sum_{n=0}^\infty a_n z^n$ and $g(z) = \sum_{n=0}^\infty b_n z^n$ are arbitrary elements of $\mathcal{D}$.

A sequence $(z_n)_{n\geq 1}$ in $\mathbb{D}$ is called a *zero set* for $\mathcal{D}$ provided that there is an element $f \in \mathcal{D}$, $f \not\equiv 0$, such that $f(z_n) = 0$, $n \geq 1$. Since $\mathcal{D} \subset H^2(\mathbb{D})$,

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