COMPUTATION OF CAPACITY VIA QUADRATIC PROGRAMMING

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Abstract. We develop a method for computing capacity based on energy minimization. The method applies to a wide variety of capacities, including Riesz, logarithmic and hyperbolic capacities. It yields rigorous upper and lower bounds which converge to the true value of the capacity. The method is illustrated with a number of examples.

On développe une méthode de calcul de la capacité basée sur la minimisation de l'énergie. La méthode s'applique à une gamme de capacités qui inclut les capacités de Riesz et les capacités logarithmique et hyperbolique. Elle fournit des bornes supérieures et inférieures rigoureuses qui convergent vers la vraie valeur de la capacité. On donne plusieurs exemples pour illustrer la méthode.

1. Introduction

In this article we develop and implement a method for computing the capacity of general compact sets. The method has the following features:

- It covers a variety of different types of capacity, including Riesz, logarithmic and hyperbolic capacities.
- It furnishes rigorous upper and lower bounds for the capacity.
- It always converges, in the sense that these bounds can be made as close as we please.
- It is systematic: the same method works for every compact set.
- It is computationally practicable.
- It has good monotonicity properties, which permit the use of extrapolation techniques to accelerate convergence.

None of the other methods of computation of capacity known to us have all these advantages. The three principal techniques of which we are aware are:

(1) Computation of Green's functions. Rostand [12] developed a technique which yields rigorous upper and lower bounds for the logarithmic capacity of 'smooth' plane sets (sets with finitely many components which are equal to the closure of their interior). However, there was no proof of convergence. Also, as a by-product of their techniques for numerical Schwarz–Christoffel mappings, Embree and Trefethen [3] were able to compute the logarithmic capacity of polygonal sets symmetric with respect to the real axis.

(2) Method of Leja points. This is a method developed by Leja [9] for logarithmic capacity in the plane and Górski [6] for Newtonian capacity in space. The idea is to construct a naïve approximation to a Fekete system (i.e. system of points maximizing the n-th diameter). Like Fekete systems, Leja systems always converge to the equilibrium measure, and can therefore be used to estimate capacity, but this method does not yield rigorous bounds.

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