ADDITIVE MAPS PRESERVING THE REDUCED MINIMUM MODULUS OF BANACH SPACE OPERATORS

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ABSTRACT. Let $\mathscr{B}(X)$ be the algebra of all bounded linear operators on an infinite dimensional complex Banach space X. We prove that an additive surjective map φ on $\mathscr{B}(X)$ preserves the reduced minimum modulus if and only if either there are bijective isometries $U : X \to X$ and $V : X \to X$ both linear or both conjugate linear such that $\varphi(T) = UTV$ for all $T \in \mathscr{B}(X)$, or X is reflexive and there are bijective isometries $U : X^* \to X$ and $V : X \to X^*$ both linear or both conjugate linear such that $\varphi(T) = UT^*V$ for all $T \in \mathscr{B}(X)$. As immediate consequences of the ingredients used in the proof of this result, we get the complete description of surjective additive maps preserving the minimum, the surjectivity and the maximum moduli of Banach space operators.

1. INTRODUCTION

Several results on linear preservers have been extended to the setting of additive preservers, and, in many cases, their extensions demonstrated to be nontrivial as the forms of additive preservers are some time not "nice" as the ones of the corresponding linear preservers. In [21], Omladič and Šemrl characterized surjective additive maps preserving the spectrum of bounded linear operators on complex Banach spaces and showed that such maps are of standard forms. This is an extension of the result due to Jafarian and Sourour [17] that describes linear spectrum-preserving maps. In [2], Bai and Hou considered a more general situation and characterized surjective additive maps preserving the spectral radius of Banach space operators, extending the result due to Brešar and Šemrl [9] from the linear setting. For further results on additive preserver problems, we refer the interested reader, for example, to [2, 3, 10, 14, 15, 16, 20, 21, 26] and the references therein.

Recently, Mbekhta described unital surjective linear maps on $\mathscr{B}(H)$, the algebra of all bounded linear operators on an infinite dimensional complex Hilbert space H, preserving several spectral quantities such as the minimum, the surjectivity and the reduced minimum moduli; see [22, 23]. In [23], he showed that a unital surjective linear map on $\mathscr{B}(H)$ preserves the reduced minimum modulus if and only if it is an isometry and conjectured that the same result remains true for the nonunital linear case. Mbekhta's articles [22] and [23], which were followed quickly by several papers treating related problems, contain several good

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