

ON THE BROWN–SHIELDS CONJECTURE FOR CYCLICITY IN THE DIRICHLET SPACE

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ABSTRACT. Let \mathcal{D} be the Dirichlet space, namely the space of holomorphic functions on the unit disk whose derivative is square-integrable. We establish a new sufficient condition for a function $f \in \mathcal{D}$ to be *cyclic*, i.e. for $\{pf : p \text{ a polynomial}\}$ to be dense in \mathcal{D} . This allows us to prove a special case of the conjecture of Brown and Shields that a function is cyclic in \mathcal{D} iff it is outer and its zero set (defined appropriately) is of capacity zero.

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