Commuting holomorphic maps on the spectral unit ball

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Abstract

We prove that if F is a holomorphic map from the open spectral unit ball of a primitive Banach algebra into itself satisfying $F(0) = 0$, $F'(0) = I$ and $F(x)x = xF(x)$ for every x, then F is the identity map. Using this, we prove that if $\mathcal{A}$ is a semisimple Banach algebra and $\mathcal{B}$ is a primitive Banach algebra, then any unital spectral isometry from $\mathcal{A}$ onto $\mathcal{B}$ which locally preserves commutativity is a Jordan morphism. The same is true when $\mathcal{A}$ and $\mathcal{B}$ are both assumed to be von Neumann algebras.