

T. Ransford, **On pseudospectra and power growth**, *SIAM J. Matrix Anal. Appl.*, 29 (2007), 699–711.

**Abstract**

The celebrated Kreiss matrix theorem is one of several results relating the norms of the powers of a matrix to its pseudospectra (i.e. the level curves of the norm of the resolvent). But to what extent do the pseudospectra actually *determine* the norms of the powers? Specifically, let  $A, B$  be square matrices such that, with respect to the usual operator norm  $\|\cdot\|$ ,

$$(*) \quad \|(zI - A)^{-1}\| = \|(zI - B)^{-1}\| \quad (z \in \mathbf{C}).$$

Then it is known that  $1/2 \leq \|A\|/\|B\| \leq 2$ . Are there similar bounds for  $\|A^n\|/\|B^n\|$  for  $n \geq 2$ ? Does the answer change if  $A, B$  are diagonalizable? What if  $(*)$  holds, not just for the norm  $\|\cdot\|$ , but also for higher-order singular values? What if we use norms other than the usual operator norm? The answers to all these questions turn out to be negative, and in a rather strong sense.