

# A SHARP FORM OF THE CRAMÉR–WOLD THEOREM

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ABSTRACT. The Cramér–Wold theorem states that a Borel probability measure  $P$  on  $\mathbb{R}^d$  is uniquely determined by its one-dimensional projections. We prove a sharp form of this result, addressing the problem of how large a subset of these projections is really needed to determine  $P$ . We also consider extensions of our results to measures on a separable Hilbert space.

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