A. Bourhim and T. J. Ransford, **Additive maps preserving local spectrum**, *Integral Equations Operator Theory*, 55 (2006), 377–385.

Abstract
Let X be a complex Banach space, and let $\mathcal{L}(X)$ be the space of bounded operators on X. Given $T \in \mathcal{L}(X)$ and $x \in X$, denote by $\sigma_T(x)$ the local spectrum of T at x.
We prove that if $\Phi : \mathcal{L}(X) \to \mathcal{L}(X)$ is an additive map such that

$$\sigma_{\Phi(T)}(x) = \sigma_T(x) \qquad (T \in \mathcal{L}(X),\ x \in X),$$

then $\Phi(T) = T$ for all $T \in \mathcal{L}(X)$. We also investigate several extensions of this result to the case of $\Phi : \mathcal{L}(X) \to \mathcal{L}(Y)$, where $X \neq Y$.
The proof is based on elementary considerations in local spectral theory, together with the following local identity principle: given $S, T \in \mathcal{L}(X)$ and $x \in X$, if $\sigma_{S+R}(x) = \sigma_{T+R}(x)$ for all rank one operators $R \in \mathcal{L}(X)$, then $Sx = Tx$.