

M. Roy and M. Urbański, **Regularity properties of hausdorff dimension in infinite conformal IFS**, *Ergodic Theory and Dynamical Systems*, 25 (2005), 1961–1983.

### **Abstract**

This paper deals with families of conformal iterated function systems (CIFS). The space of all CIFS, with common seed space  $X$  and alphabet  $I$ , is successively endowed with the topology of pointwise convergence and a new, weaker topology called  $\lambda$ -topology. It is proved that the pressure and the Hausdorff dimension of the limit set are continuous with respect to the topology of pointwise convergence when  $I$  is finite, and are lower semi-continuous, though generally not continuous, when  $I$  is infinite. It is then shown that these two functions are, in any case, continuous in the  $\lambda$ -topology. The concepts of analytic, regularly analytic and plane-analytic families of CIFS are also introduced. It is established that if a family of CIFS is regularly analytic, then the Hausdorff dimension function is real-analytic; if a family is plane-analytic, then the Hausdorff dimension function is continuous and subharmonic, though not necessarily real-analytic. These results are then applied to finite parabolic CIFS. Counterexamples highlighting breakdowns of real-analyticity in the Hausdorff dimension among analytic, but not regularly analytic, families are further provided. Such families often exhibit a phenomenon coined phase transitions. Sufficient conditions preventing the occurrence of such transitions are supplemented.