T. Ransford and M. Valley, **Subharmonicity in von Neumann algebras**, *Studia Math.*, 170 (2005), 219–226.

Abstract
Let $\mathcal{M}$ be a von Neumann algebra with unit $1_{\mathcal{M}}$. Let τ be a faithful, normal, semifinite trace on $\mathcal{M}$. Given $x \in \mathcal{M}$, denote by $\mu_t(x)_{t\geq 0}$ the generalized s-numbers of x, defined by

$$\mu_t(x) = \inf\{\|xe\| : e \text{ is a projection in } \mathcal{M} \text{ with } \tau(1_{\mathcal{M}} - e) \leq t\} \quad (t \geq 0).$$

We prove that, if D is a complex domain and $f : D \to \mathcal{M}$ is a holomorphic function, then, for each $t \geq 0$,

$$\lambda \mapsto \int_0^t \log \mu_s(f(\lambda))\, ds$$

is a subharmonic function on D. This generalizes earlier subharmonicity results of White and Aupetit on the singular values of matrices.