F. Gourdeau, Z. A. Lykova and M. C. White, **A Künneth formula in topological homology and its applications to the simplicial cohomology of $\ell^1(\mathbf{Z}_+^k)$**, *Studia Math.*, 166 (2005), 29–54.

Abstract

For the semigroup algebra $\ell^1(\mathbf{Z}_+^k)$, we show that the simplicial cohomology groups $\mathcal{H}^n(\ell^1(\mathbf{Z}_+^k), \ell^1(\mathbf{Z}_+^k)')$ are Banach spaces and we describe them explicitly. To establish these descriptions, we obtain a Knneth formula for some chain complexes in the categories of Fréchet and Banach spaces which enables us to calculate the simplicial homology groups $\mathcal{H}_n(\ell^1(\mathbf{Z}_+^k), \ell^1(\mathbf{Z}_+^k))$ of $\ell^1(\mathbf{Z}_+^k)$. We consider a complex $\mathcal{X}$ of Banach spaces and continuous boundary maps d_n with closed ranges and prove that $H^n(\mathcal{X}') \cong H_n(\mathcal{X})'$, where $H_n(\mathcal{X})'$ is the dual space of the homology group of $\mathcal{X}$ and $H^n(\mathcal{X}')$ is the cohomology group of the dual complex $\mathcal{X}'$. A Knneth formula for chain complexes of nuclear Fréchet spaces and continuous boundary maps with closed ranges is also obtained.