

J. Mashreghi and T. J. Ransford, **Binomial sums and functions of exponential type**, *Bull. London Math. Soc* 37 (2005), 15–24.

**Abstract**

Let  $(a_n)_{n \geq 0}$  be a sequence of complex numbers, and, for  $n \geq 0$ , let

$$b_n = \sum_{k=0}^n \binom{n}{k} a_k \quad \text{and} \quad c_n = \sum_{k=0}^n \binom{n}{k} (-1)^{n-k} a_k.$$

We prove a number of results relating the growth of the sequences  $(b_n)$  and  $(c_n)$  to that of  $(a_n)$ . For example, given  $p \geq 0$ , if  $b_n = O(n^p)$  and  $c_n = O(e^{\epsilon\sqrt{n}})$  for all  $\epsilon > 0$ , then  $a_n = 0$  for all  $n > p$ . Also, given  $0 < \rho < 1$ , we have  $b_n, c_n = O(e^{\epsilon n^\rho})$  for all  $\epsilon > 0$  iff  $n^{1/\rho-1}|a_n|^{1/n} \rightarrow 0$ . We further show that, given  $\beta > 1$ , if  $b_n, c_n = O(\beta^n)$ , then  $a_n = O(\alpha^n)$ , where  $\alpha = \sqrt{\beta^2 - 1}$ , thereby proving a conjecture of Chalendar, Kellay and Ransford.

The principal ingredients of the proofs are a Phragmén–Lindelöf theorem for entire functions of zero exponential type, and an estimate for the expected value of  $e^{\phi(X)}$ , where  $X$  is a Poisson random variable.