**Abstract**

Given a completely non unitary contraction $T$, some necessary (and, in certain cases, sufficient) conditions are found for the range of the $H^\infty$ calculus, $H^\infty(T)$, and the commutant, $\{T\}'$, to contain non-zero compact operators, and for the finite rank operators of $\{T\}'$ to be dense in the set of compact operators of $\{T\}'$. A sufficient condition is given for $\{T\}'$ to contain non-zero operators from the Schatten-von Neumann classes $S_p$. The proofs are mostly based on the Sz-Nagy and Foias functional model and on the link between operators from the commutant and Hankel operators.