C. Costara, **The symmetrized bidisc as a counterexample to the converse of Lempert's theorem**, *Bull. London Math. Soc.*, 36 (2004), 656–662.

Abstract
Let $G \subset \mathbf{C}^2$ be the open symmetrized bidisc, namely

$$G = \{(\lambda_1 + \lambda_2, \lambda_1\lambda_2) : |\lambda_1| < 1, |\lambda_2| < 1\}.$$

We prove that G is not biholomorphic to any convex domain in $\mathbf{C}^2$. By combining this result with earlier work of Agler and Young, we obtain that G is a bounded domain on which the Carathéodory distance and the Kobayashi distance coincide, and yet, not biholomorphic to a convex set.