O. El-Fallah and T. J. Ransford, **Peripheral point spectrum and growth of powers of operators**, *J. Operator Theory*, 52 (2004), 89–101.

Abstract

Let E be a closed subset of the unit circle. A result of Nikolski shows that, if T is an operator on a separable Hilbert space whose point spectrum contains E, and if $0 < \alpha < \dim_H E$ (the Hausdorff dimension of E), then

$$\sum_n n^{\alpha-1}\|T^n\|^{-2} < \infty.$$

We complement this result by showing that, for each $\beta > \overline{\dim}_B E$ (the upper box dimension of E), there exists an operator T on a separable Hilbert space, whose point spectrum contains E, and such that

$$\sum_n n^{\beta-1}\|T^n\|^{-2} = \infty.$$

We also prove some more refined results along the same lines.