K. Kellay, **Fonctions intérieures et vecteurs bicycliques**, *Arch. Math. (Basel)*, 77 (2001), 253–264.

Abstract
We consider weights ω on $\mathbf{Z}$ such that $\omega(n) \to 0$ as $n \to +\infty$, $\omega(n) \to +\infty$ as $n \to -\infty$, and satisfying some regularity conditions. Denote by Γ the unit circle, set

$$L^2_\omega = \left\{ f \in L^2(\Gamma) : ||f||_\omega = \left(\sum_{n \in \mathbf{Z}} |\hat{f}(n)|^2 \omega(n)^2 \right)^{1/2} < +\infty \right\},$$

and denote by $S_\omega : f(e^{it}) \to e^{it} f(e^{it})$ the usual shift on L^2_ω. We show that if

$$\sum_{n \geq 1} \frac{n}{\ln \omega(-n)} (2\omega(n)^{-2} - \omega(n-1)^{-2} - \omega(n+1)^{-2}) < +\infty,$$

then there exists a singular inner function U not bicyclic in L^2_ω, that is, the closure of $\mathrm{Span}\{S^n_\omega U : n \in \mathbf{Z}\}$ is a proper subspace of L^2_ω.