B. J. Cole and T. J. Ransford, **Jensen measures and harmonic measures**, *J. Reine Angew. Math. (Crelle)* 541 (2001), 29–53.

Abstract
Let Ω be an open subset of $\mathbf{R}^d$ ($d \geq 2$) and let $x \in \Omega$. Denote by $J_x(\Omega)$ the family of Jensen measures for x on Ω, and by $H_x(\Omega)$ the subclass consisting of harmonic measures at x for domains $D \subset\subset \Omega$. Our main result (Theorem 1.3) is that, given a function $\phi\colon \Omega \to [-\infty, \infty)$, universally measurable and locally bounded above,

$$\inf\Big\{\int \phi\, d\mu : \mu \in J_x(\Omega)\Big\} = \inf\Big\{\int \phi\, d\omega : \omega \in H_x(\Omega) \cup \{\delta_x\}\Big\}.$$

This allows us to re-express recent results about the duality between subharmonic functions and Jensen measures in terms of the more concrete class of harmonic measures. As applications, we obtain a converse to the maximum principle applicable to bounded Borel functions (Theorem 1.4), as well as information regarding extreme Jensen measures (Theorem 1.5). We conclude by relating these ideas to recent work of a number of authors on analytic-disk measures.