

B. Aupetit, **Spectrum-preserving linear mappings between Banach algebras or Jordan-Banach algebras**, *J. London Math. Soc.*, (2) 62 (2000), no. 3, 917-924.

**Abstract**

Spectrum-preserving linear mappings were studied for the first time by G. Frobenius [18]. He proved that a linear mapping  $\Phi$  from  $M_n(C)$  onto  $M_n(C)$  which preserves the spectrum has one of the forms  $\Phi(x) = axa^{-1}$  or  $\Phi(x) = a^t x a^{-1}$ , for some invertible matrix  $a$ . (Incidentally the hypothesis that  $\Phi$  is onto is superfluous; see Proposition 2.1(i).) This result was extended by J. Dieudonn [17] supposing  $\Phi$  onto and satisfying  $\text{Sp } \Phi(x) \subset \text{Sp } x$ , for every  $n \times n$  matrix  $x$ .

Several results of M. Nagasawa, S. Banach and M. Stone, R. V. Kadison, A. Gleason and J. P. Kahane and W. Zelazko led I. Kaplansky in [22] to the following problem: given two Banach algebras with unit and  $\Phi$  a linear mapping from  $A$  into  $B$  such that  $\Phi(1) = 1$  and  $\text{Sp } \Phi(x) \subset \text{Sp } x$ , for every  $x \in A$ , is it true that  $\Phi$  is a Jordan morphism? With this general formulation, this question cannot be true (see [2], p. 28).