I. Chalendar, K. Kellay and T. J. Ransford, **Binomial sums, moments and invariant subspaces**, *Israel J. Math.*, 115 (2000), 303-320.

Abstract

The main result of this paper is that if a sequence of complex numbers $(a_n)_{n\geq 0}$ satisfies

$$\sum_{\substack{k=0 \\ k \text{ even}}}^{n} \binom{n}{k} a_k = O(n^r) \qquad \text{and} \qquad \sum_{\substack{k=0 \\ k \text{ odd}}}^{n} \binom{n}{k} a_k = O(n^r) \qquad \text{as } n \to \infty,$$

for some integer $r \geq 0$, then $a_n = 0$ for all $n > r$. As an application, we deduce a localized form of a theorem of Allan about nilpotent elements in Banach algebras, and this in turn leads to an invariant-subspace theorem. As a further application, we prove a variant of Carleman's theorem on the unique determination of probability distributions by their moments. The paper concludes with a quantitative form of the main result.