Abstract

Let $\mathcal{M}_n$ denote the space of complex $n \times n$ matrices, and let Ω_n denote the spectral unit ball of $\mathcal{M}_n$, namely the set of matrices in $\mathcal{M}_n$ whose eigenvalues lie within the open unit disc. As a step towards the eventual classification of the holomorphic automorphisms of Ω_n, we prove that every such automorphism F satisfying $F(\mathbf{0}) = \mathbf{0}$ and $F'(\mathbf{0}) = I$ has the property that $F(\mathbf{x})$ is conjugate to $\mathbf{x}$ for each $\mathbf{x} \in \Omega_n$. This result is obtained by combining earlier work of Ransford and White with a general theorem about spectrum-preserving maps proved in this paper.