G. A. Philippin and S. Vernier Piro, **Explicit exponential decay bounds in quasi-linear parabolic problems**, *J. Inequal. Appl.*, 3 (1999), no. 1, 1–23.

Abstract

We study the ground water flow described by the following equations

$$\Delta G(u) + f(u) = u_t, \ x \in \Omega, \ t > 0 \tag{1}$$

where Ω is a bounded one-dimensional domain and $f(u)$ is the source term. We note that this equation for the particular choice of $G(u) = u^m$ is the well known porus media equation.

We derive a maximum principle for a particular class of functionals defined on strong solutions of (1)

$$\Psi(u, u_x^2) := (G(u_x^2) + \alpha u^2 + 2F(u)) \exp(2\alpha\beta t)$$

with $F(u) := \int_0^u f(s)\,ds$, where α is an arbitrary parameter ≥ 0 and β is some positive constant to be determined. We compute a range of values of α for which Ψ takes its maximum value initially, i.e. at $t = 0$ if $f(u) = 0$. If $f(u) \neq 0$, the solution u may blow-up at some time; so we establish conditions on data sufficient to prevent blow-up for u and even sufficient to obtain its exponential decay in time.