A. Henrot and G. A. Philippin, **Some overdetermined boundary value problems with elliptical free boundaries**, *SIAM J. Math. Anal.*, 29 (1998), no. 2, 309–320.

Abstract
In this paper we study three different overdetermined boundary value problems in R^2: a problem of torsion, a problem of electrostatic capacity, and a problem of polarization. In each case we prove that a solution exists if and only if the free boundary is an ellipse. The techniques we use rely on classical complex function theory, maximum principle, and some topological argument.