

F. Gourdeau, **Amenability and the second dual of a Banach algebra**, *Studia Math.*, 125:1 (1997), 75–81.

Abstract

The study of cohomological properties of \mathcal{A}^{**} in relation to those of \mathcal{A} goes back to B. E. Johnson's seminal article. Recently, Gharahmani, Loy and Willis have studied the amenability and weak amenability of \mathcal{A} in relation to the same properties for \mathcal{A}^{**} , with an emphasis on the Banach algebra $L^1(\mathcal{G})$. One of their result is that the amenability of \mathcal{A}^{**} implies the amenability of \mathcal{A} : this result was originally proved in the author's PhD dissertation by other methods, but had not been published.

In this article, we show how Arens' construction of a product on the second dual of a Banach algebra enables us to extend derivations from \mathcal{A} into a bimodule \mathcal{X} to derivations from \mathcal{A}^{**} into \mathcal{X}^{**} , answering a question raised in B. E. Johnson's memoir. This is then used, along with a criterion for amenability which does not involve duals, to give a simple proof that \mathcal{A}^{**} amenable implies \mathcal{A} amenable.